The explanation of unexpected temperature dependence of the muon catalysis in solid deuterium

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Abstract

It is shown that due to the smallness of the inelastic cross-section of the $d\mu$-atoms scattering in the crystal lattice at sufficiently low temperatures the $dd\mu$-mesomolecules formation from the upper state of the hyperfine structure $d\mu(F = 3/2)$ starts earlier than the mesoatoms thermolization. It explains an approximate constancy of the $dd\mu$-mesomolecule formation rate in solid deuterium.

Highly effective resonant mechanism of the $dd\mu$ mesomolecule formation followed by the $dd \rightarrow^3 He + n$ or $dd \rightarrow T + p$ nuclear synthesis becomes possible in the gaseous and liquid deuterium due to the presence of the oscillatory-rotary level ($\nu = 1$, $K = 1$) in the $dd\mu$ mesomolecule, which has a small binding energy [1-3]. The matter is that the binding energy of the $dd\mu$ mesomolecule in this state, ($\epsilon \approx 2$ eV), is little bit less than that which is needed to excite the oscillatory level $n = 7$ of the $[d\mu d, d]2e$ molecular complex, which arises when one of the nuclei of the D_2 molecule is substituted by the $dd\mu$ mesomolecule. The lack of the energy is filled up by the thermal motion energy of the $d\mu$-mesoatom thermolized in the matter, which collides with one of the D_2-molecule deutrons, that allows to fulfill the condition of the resonant formation [4].

The presence of two hyperfine structure levels with the total spin $F = 3/2$ and $F = 1/2$ in the $d\mu$-mesoatoms (the energy difference between them is equal to $\Delta E = 0.0485$ eV) requires a more detailed investigation of the resonant mechanism. One should take into account the fact that the $dd\mu$-mesomolecule produced in the rotatory state $K = 1$ with total nuclei spin equal to 1 via the resonant mechanism also has two hyperfine structure levels with the total spin $S = 3/2$ and $S = 1/2$, which have the energy difference $\Delta E =$

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0.242 eV. (the splitting of these levels caused by the mesomolecule rotation is unimportant and can be neglected in the first approximation).

Analysing the time-depedence of the muon catalysis at different temperatures one can determine the rate of the mesomolecules formation from different states of the hyperfine structure of the $d\mu$-mesoatom as well as the transition rate from upper level with the spin $F = 3/2$ to lower level with the spin $F = 1/2$, which takes place in exchange collisions of the $d\mu$-mesoatom with the deuterium nuclei [5]. According to the theoretical calculations [6] the rate of the resonant formation of the $dd\mu$-molecule from the lowest state of the $d\mu(F = 1/2)$ mesoatom hyperfine structure becomes smaller when the temperature of the gas matter or liquid is decreased, since thermal energies of the thermolized d_μ-mesoatoms are insufficient to fulfill the resonance condition. At $T \approx 70^o$ K the resonance mechanism of the $dd\mu$-mesomolecules formation for the $d_\mu(F = 1/2)$-state is almost completely ruled out and the $dd\mu$-mesomolecule formation from the $F = 1/2$-state is solely induced by the nonresonant mechanism, in which the mesomolecule binding energy is carried off by atomic electron [7-11] (see Fig. 1).

The probability of the nonresonant transition is about two order of magnitude less than that for resonant one, and this mechanism leads to the formation of the mesomolecules in the ground states with even rotatory numbers [2] and even total nuclei spin $I = 2, 0$, and, in part, in the ground rotatory state with $K = 1, \quad \nu = 0$. The observed change of the ratio of the dd-reaction channels, $R = \frac{\Gamma(dd \to {}^3He+n)}{\Gamma(dd \to T+p)}$ [12], which occurs in the μ-catalysis from the $F = 1/2$ state when the temperature is decreased (that is due to the difference of this ratio for reactions starting from the rotatory state $K = 1$ with total nuclei spin $I = 1$ and from the S-state with even rotatory numbers and spins $I = 0, \ I = 2$), is considered as an additional experimental proof.

As for the process of the $dd\mu$-molecule formation from the $d_\mu(F = 3/2)$-state, its rate weakly depends on the temperature and starts to decrease at $T \simeq 26^o$K only. The temperature dependence of the molecule formation rate measured in the dense gas and in liquid deuterium [13,14] agreed well with the theoretical calculations [6] (see. Fig. 2). However, the data on the $dd\mu$-mesomolecule formation rate in the solid hydrogen at $T = 3^o$ K [15] exceeds the theoretical expectations by more than order of magnitude. The detailed studies conducted on the same setup in the solid deuterium and liquid [16] in the temperature range of $5 - 50$ have confirmed this result (see Fig. 2). It turned out that at temperatures below $T \simeq 20^o$K the rate of the resonant formation of the $dd\mu$-mesomolecules from the $d\mu(F = 3/2)$-state does not change practically. Analogously, in agreement with the theory, the rate of the nonresonant formation from the $d\mu(F = 1/2)$-state does not change too, and what is more important the rate of the transition between hyperfine structure levels of the mesoatoms, $d_\mu(F = 3/2) \to d_\mu(F = 1/2)$ ($\lambda_d \approx 3.1 \cdot 10^7 \mathrm{c}^{-1}$) does not change too. Thus, the paradoxical situation occurs. It seems as if the $d\mu$-mesoatoms in the hyperfine structure state $F = 3/2$ would conserve their energy and would not be thermolized with the temperature decrease below 20^oK. In this paper I would like to point out that such phenomenon can be explained in a natural way in terms of the type of energy loss by slow d_μ-atoms in the crystal lattice and it is almost independent on the lattice structure itself.

Due to the electroneutrality an smallness of the d_μ-mesoatoms their scattering in the crystal is analogous, in a large extent, to the slow neutron scattering. As it was shown by I.Ya. Pomeranchuk in 1937 [17], the neutrons at sufficiently low temperatures occurs, in bulk, scatters elastically on whole crystal lattice and this process does not accompanied by the energy loss (one could only regret that having obtained this result, I.Ya. Pomeranchuk had not predict the Mössbauer effect). As for inelastic collisions, they are connected with the one-photon lattice excitation, and the mean free path, l_{in}, is greater than elastic one l_{el}. According to [17], for identical nuclei (when the neutron wave length becomes smaller than the lattice constant) one gets

$$l_{in} = \frac{7}{8} l_{el} \left(\frac{k_B \Theta}{E} \right)^3 , \tag{1}$$

where E is the neutron energy, Θ is the Debey temperature. At $E << k_B\Theta$ one has $l_{in} >> l_{el}$. Besides, if the length of the neutron capture $l_c = \frac{1}{N\sigma_c}$ (where σ_c is the capture cross-section, N is the nuclei concentration) will be significantly smaller than l_{in}, the neutron deceleration is ceased and they will be captured by nuclei before their thermolization occurs. Analogous arguments can be applied to the case of the $d\mu$-mesoatoms behaviour in crystal lattice.

The cross-section of the elastic scattering of slow $d\mu$-atoms in the hyperfine structure state $F = 3/2$ on deutons has the form [5]:

$$\sigma_{3/2\to 3/2} = 4\pi \left\{ \frac{1}{2}\lambda^2 g + \frac{1}{3}\left(\frac{\lambda g + 5\lambda u}{6} \right)^2 + \frac{1}{6}\left(\frac{\lambda g + \lambda u}{3} \right)^2 \right\}, \tag{2}$$

where λ_g and λ_u are the $d\mu$ scattering lengths in potentials V_g and V_u, which correspond to muon molecular orbits Σ_g and Σ_u in the filed of two deutons (where the first order corrections over the m_μ/M_d mass ratio due to the motion nonadiabaticity are taken into account) [5]. The cross-section of the $d\mu(F = 3/2) \to d_\mu(F = 1/2)$ transition due to collisions with muon exchange is as follows

$$\sigma_{3/2\to 1/2} = \frac{\pi}{3}(\lambda_g - \lambda_u)^2 \frac{k_0}{k_1}, \tag{3}$$

where $k_1 = \sqrt{\frac{M_d E}{\hbar}}$; $k_0 = \sqrt{\frac{M_d(E+\Delta E)}{\hbar}}$, E is the $d\mu(F = 3/2)$ mesonucleus energy , $\Delta E = 0.0485$ eV is the energy of the hyperfine splitting in the $d\mu$-atom. The rate of the transition into the lowest hyperfine structure state is [5]

$$\lambda_d = N\sigma_{3/2\to 1/2} \cdot v_1 = \frac{\pi}{3}(\lambda_g - \lambda_u)^2 N \cdot v_0; \tag{4}$$

$$v_0 = 2\sqrt{\frac{\Delta E}{Md}} \simeq 3 \cdot 10^5 \text{cm/c}.$$

Using the experimental value of $\lambda_d \simeq 3,1 \cdot 10^7$ at $N = 4 \cdot 25 \cdot 10^{22}\text{cm}^{-3}$ one gets $|\lambda_g - \lambda_u| = 1,87\ a_\mu \left(a_\mu = \frac{\hbar^2}{m_\mu e^2} = 2,56 \cdot 10^{-11}\text{cm} \right)$. The estimates [5] show that scattering

lengths λ_g and λ_u are close to each other, and their average value is about $\bar{\lambda} = (5 \div 6)a_\mu$. Thus, elastic cross-section (2) can be rewritten in the following form

$$\bar{\sigma}_{el} = 4\pi\bar{\lambda}^2 a_\mu^2. \tag{5}$$

Mean free path, connected with $dd\mu$-mesomolecule formation, is equal to:

$$l_{dd\mu} = \frac{1}{N\sigma_{dd\mu}} = \frac{v}{\lambda_{dd\mu}}, \tag{6}$$

where $\sigma_{dd\mu}$ is the effective cross-section of mesomolecule formation, v is the $d\mu$-mesoatoms velocity, which under the thermolization condition is equal to $v = \left(\frac{3k_BT}{M_d}\right)^{1/2}$. From the condition $l_{dd\mu} < l_{in}$ and equations (1) and (5) one can estimate the temperature, at which the mesomolecules formation starts early than mesoatoms thermolization:

$$T \leq 0.4\Theta_d^{6/7} \left(\frac{M_d}{3k_B}\right)^{1/7} \left(\frac{\lambda_{dd\mu}}{\bar{\lambda}^2 a_\mu^2 N}\right)^{2/7} \tag{7}$$

For the experimental value $\lambda dd\mu \simeq 2.27 \cdot 10^6 1/\mathrm{s}$, using $\bar{\lambda} \sim 6a_\mu$ and assuming the Debey temperature for deuterium equal to $\Theta_d = 74^o\mathrm{K}$, one gets

$$T \leq 12.5^o\mathrm{K}, \tag{8}$$

that, even being roughly estimated, agrees well with the data [15,16].

Thus, in the solid deuterium the resonant formation of the $dd\mu$-mesomolecules from upper level of the $d\mu$-mesoatom hyperfine structure starts before the $d\mu$-atom thermolization. Namely this phenomenon explains the independence of muon catalysis in sold deuterium on the temperature. It should be noted, that ephythermal formation of the $dd\mu$-mesomolecules has been observed earlier in the matter, composed of the HD-molecules [18]. It was explained by the fact that the effective cross-section of the $d\mu$-mesoatoms scattering on protons is small due to the Ramsauer effect [19], and, thus, in the process of the $d\mu$-mesoatoms deceleration their energies with particular probability have the values, which correspond to the region of the resonant formation on higher oscillatory states of the $[dd\mu, d]2e$ complex.

At same time, the crystal lattice in solid deuterium slightly effects on the rate of the resonant formation of the $dt\mu$-mesomolecules due to the difference in locations and widths of the resonant level for the $dt\mu$ and $dd\mu$ mesomolecules [20].

As I found out after this paper has been written, L.I. Ponomarev, basing on the experimental data analysis, has suggested the hypothesis that the thermolization of the $d\mu$-atoms does not occur in solid deuterium the experimental data analysis. I sincere thank L.I. Ponomarev and G.G. Semenchuk for fruitful discussions. This work was supported, in part, by the RFBR under grants 99-02-16558 and 00-15-96645.

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Figure captions

1. Molecular formation rates $\tilde{\lambda}_{1/2}$ and $\tilde{\lambda}_{3/2}$ from all experiments performed at PSI and fits done in [21].

2. Experimental dependence of the deuterium mesomolecule formation rate on the temperature and spin state of the deuterium mesoatom. The data points are marked as follows: circles represent the experimental data [2], square boxes – the data from [15], triangle boxes – the data from [14], crossed points – the data from [13], solid line – theoretical predictions [6].

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